

TDC Odd Semester Exam., 2020
held in July, 2021

PHYSICS
(Honours)

(3rd Semester)

Course No. : PHS-303

(Mathematical Physics—II)

Full Marks : 35
Pass Marks : 12

Time : 2 hours

*The figures in the margin indicate full marks
for the questions*

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. (a) Explain with example, what are 'order' and 'degree' of a differential equation. What is a singular point? 2+1=3
- (b) Solve the following differential equation : 4

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$$

2. (a) Write the condition when a differential equation is homogeneous. 1
- (b) Using Frobenius method, solve the following differential equation : 6

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} - xy = 0$$

UNIT—II

3. Prove the following :
- (i) $n P_n = (2n-1)x P_{n-1} - (n-1)P_{n-2}$
- (ii) $(2n-1)P_n = P_{n-1}' - P_{n-1}$
- where P_n represents Legendre polynomials. 4+3=7

4. (a) Prove the Rodrigues' formula
- $$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n \quad 6$$
- (b) Write the generating function for Legendre polynomial. 1

(3)

UNIT—III

5. For Bessel's function $J_n(x)$ prove the recurrence relation : 3½+3½=7

(i) $J_{n-1}(x) - J_{n+1}(x) = \frac{2n}{x} J_n(x)$

(ii) $J_{n-1}(x) + J_{n+1}(x) = 2J_n(x)$

6. (a) Prove

$$J_{1/2}(x) = \sqrt{\frac{2}{x}} \sin x \quad 5$$

- (b) Write the Bessel's function of first kind. 2

UNIT—IV

7. (a) What is a tensor? What is meant by the rank of a tensor? 2+2=4

- (b) Show that the Kronecker delta δ^i_j is a mixed tensor of rank two. 3

8. (a) What are covariant and contra-variant tensors? 4

(4)

- (b) If A_{ij} and B_{ij} are two tensors, then prove that

$$A^{ij}B_{ij} = A_{ij}B^{ij} \quad 3$$

UNIT—V

9. (a) Express the complex number $\frac{2-i}{3+i}$ in polar form. 3

- (b) Explain 'neighbourhood' and 'continuity'. 2+2=4

10. (a) Explain the condition for a function to be analytic. 2

- (b) Deduce the Cauchy-Riemann conditions in complex analysis. 5
